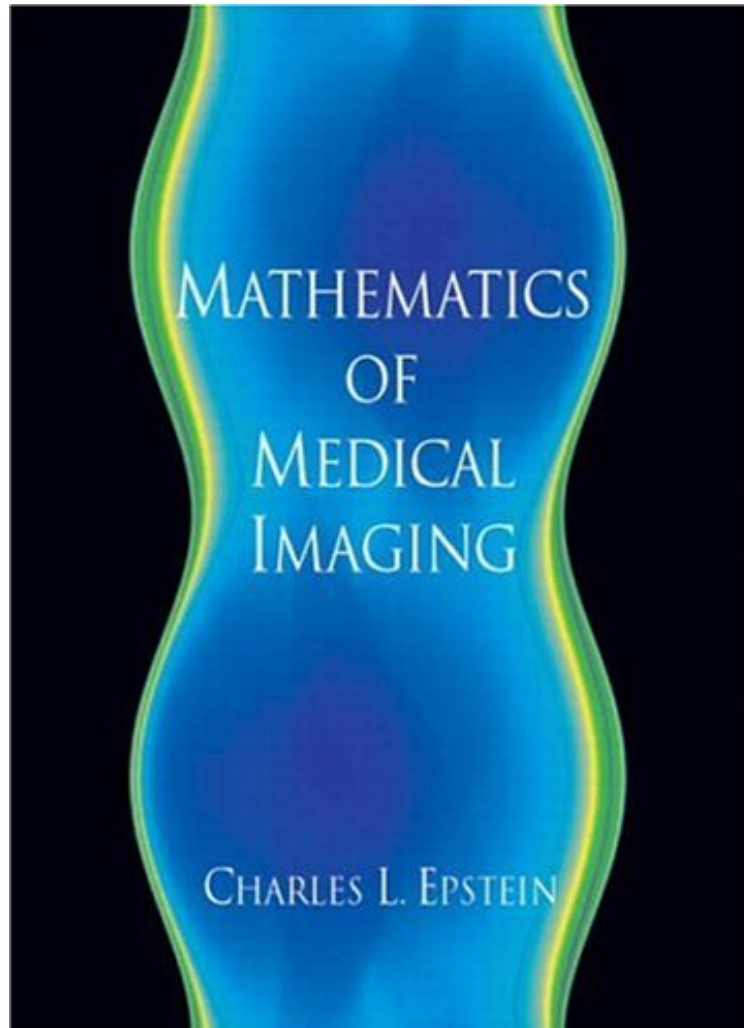


# Introduction to the Mathematics of Medical Imaging

*Charles L. Epstein*

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This book provides an integrated presentation of mathematics and its application to problems in medical imaging. Key topics include data collection, signal processing and noise analysis. The book should be suitable for self study by a motivated person with a solid mathematical background interested in medical imaging.

From the Back Cover This book provides an integrated presentation of mathematics and its application to problems in medical imaging. Key topics include data collection, signal processing and noise analysis. The book should be suitable for self study by a motivated person with a solid mathematical background interested in medical imaging. Excerpt. Reprinted by permission. All rights reserved. Over the past several decades, advanced mathematics has quietly insinuated itself into many facets of our day-to-day life. Mathematics is at the heart of technologies from cellular telephones and satellite positioning systems to online banking and metal detectors. Arguably no technology has had a more positive and profound effect on our lives than medical imaging, and in no technology is the role of mathematics more pronounced or less appreciated. X-ray tomography, ultrasound, positron emission tomography, and magnetic resonance imaging have fundamentally altered the practice of medicine. At the core of each modality is a mathematical model to interpret the measurements and a numerical algorithm to reconstruct an image. While each modality operates on a different physical principle and probes a different aspect of our anatomy or physiology, there is a large overlap in the mathematics used to model the measurements, design reconstruction algorithms, and analyze the effects of noise. In this text we provide a tool kit, with detailed operating instructions, to work on the sorts of mathematical problems that arise in medical imaging. Our treatment steers a course midway between a complete, rigorous mathematical discussion and a cookbook engineering approach. The target audience for this book is junior or senior math undergraduates with a firm command of multivariable calculus, linear algebra over the real and complex numbers, and the basic facts of mathematical analysis. Some familiarity with basic physics would also be useful. The book is written in the language of mathematics, which, as I have learned, is quite distinct from the language of physics or the language of engineering. Nonetheless, the discussion of every topic begins at an elementary level and the book should, with a little translation, be usable by advanced science and engineering students with some mathematical sophistication. A large part of the mathematical background material is provided in two appendices. X-ray tomography is employed as a pedagogical machine, similar in spirit to the elaborate devices used to illustrate the principles of Newtonian mechanics. The physical principles used in x-ray tomography are simple to describe and require little formal background in physics to understand. This is not the case in any of the other modalities listed nor in less developed modalities like infrared imaging or impedance tomography. The mathematical problems that arise in x-ray tomography and the tools used to solve them have a great deal in common with those used in the other imaging modalities. This is why our title is Introduction to the Mathematics of Medical Imaging instead of Introduction to the Mathematics of X-Ray Tomography. A student with a thorough understanding of the material in this book should be mathematically prepared for further investigations in most subfields of medical imaging. Very good treatments of the physical principles underlying the other modalities can be found in Radiological Imaging by Harrison H. Barrett and William Swindell, 4, Principles of Computerized Tomographic Imaging by Avinash C. Kak and Malcolm Slaney, 56, Foundations of Medical Imaging by Cho, Jones, Singh, 14, Image Reconstruction from Projections by Gabor T. Herman, 35, and Magnetic Resonance Imaging by E. Mark Haacke, Robert W Brown, Michael R. Thompson, Ramesh Venkatesan, 33. Indeed these books were invaluable sources as I learned the subject myself. My treatment of many topics owes a great deal to these books as well as to the papers of Larry Shepp and Peter Joseph and their collaborators. More advanced treatments of the mathematics and algorithms introduced here can be found in The Mathematics of Computerized Tomography by Frank Natterbr, 72, and Mathematical Methods in Image Reconstruction by Frank Natterer and Frank Wubbellig, 73. The order and presentation of topics is somewhat nonstandard. The organizing principle of this book is the evolutionary development of an accurate and complete model for x-ray tomography. We start with a highly idealized mathematical model for x-ray tomography and work toward more realistic models of the actual data collected and the algorithms used to reconstruct images. After some preliminary material we describe a continuum, complete data model phrased in terms of the Radon transform. The Fourier transform is introduced as a tool, first to invert the Radon transform and subsequently for image processing. The first refinement of this model is to take account of the fact that real data are always sampled. This entails the introduction of Fourier series, sampling theory, and the finite Fourier transform. After introducing terminology and concepts from filtering theory, we give a detailed synthesis of the foregoing ideas by describing how continuum, shift invariant, linear filters are approximately implemented on finitely sampled data. With these preliminaries in hand, we return to the study of x-ray tomography, per se. Several designs for x-ray computed tomography machines are described, after which we derive the corresponding implementations of the filtered back-projection algorithm. At first we assume that the x-ray beam is one dimensional and monochromatic. Subsequently we analyze the effects of a finite width beam and various sorts of measurement and modeling errors. The last part of the book is concerned with noise analysis. The basic concepts of probability theory are reviewed and applied to problems in imaging. The notion of signal-to-noise ratio (SNR) is introduced and used to analyze the effects of quantum noise on images reconstructed using filtered back-projection. A maximum likelihood algorithm for image reconstruction in positron emission tomography is described. The final chapter introduces the idea of a random process. We describe the random processes commonly encountered in imaging and an elementary example of an optimal filter. We conclude with a brief analysis of noise in the continuum model of filtered back-projection. The book begins with an introduction to the idea of using a mathematical model as a tool to extract the physical state of system from feasible measurements. In medical

imaging, the "state of the system" in question is the anatomy and physiology of a living human being. To probe it nondestructively requires considerable ingenuity and sophisticated mathematics. After considering a variety of examples, each a toy problem for some aspect of medical imaging, we turn to a description of x-ray tomography. This leads us to our first mathematical topic, integral transforms. The transform of immediate interest is the Radon transform, though we are quickly led to the Abel transform, Hilbert transform, and Fourier transform. Our study of the Fourier transform is dictated by the applications we have in mind, with a strong emphasis on the connection between the smoothness of a function and the decay of its Fourier transform and vice versa. Many of the basic ideas of functional analysis appear as 'we consider these examples. The concept of a weak derivative, which is ubiquitous in the engineering literature and essential to a precise understanding of the Radon inversion formula, is introduced. This part of the book culminates in a study of the Radon inversion formula. A theme in these chapters is the difference between finite- and infinite-dimensional linear algebra. The next topics we consider are Fourier series, sampling, and filtering theory. These form the basis for applying the mathematics of the Fourier transform to real-world problems. Chapter 8 is on sampling theory; we discuss the Nyquist theorem, the ShannonWhittaker interpolation formula, the Poisson summation formula, and the consequences of undersampling. In Chapter 9, on filtering theory, we recast Fourier analysis as a tool for image and signal processing. The chapter concludes with an overview of image processing and a linear systems analysis of some basic imaging hardware. We then discuss the mathematics of approximating continuous time, linear shift invariant filters on finitely sampled data, using the finite Fourier transform. In Chapters 11 and 12 the mathematical tools are applied to the problem of image reconstruction in x-ray tomography. These chapters are largely devoted to the filtered back-projection algorithm, though other methods are briefly considered. After deriving the reconstruction algorithms, we analyze the point spread function and modulation transfer function of the full measurement and reconstruction process. We use this formalism to analyze a variety of imaging artifacts. Chapter 13 contains a brief description of "algebraic reconstruction techniques," which are essentially methods for solving large, sparse systems of linear equations. The final topic is noise in the filtered back-projection algorithm. This part of the book begins with an introduction to probability theory. Our presentation uses the language and ideas of measure theory, in a metaphoric rather than a technical way. Chapter 14 concludes with a study of specific probability distributions that are important in imaging. In Chapter 15 we apply probability theory to a variety of problems in medical imaging. This chapter includes the famous resolution-dosage fourth power relation, which shows that to double the resolution in a CT image, keeping the SNR constant, the radiation dosage must be increased by a factor of 16! The chapter ends with an introduction to positron emission tomography and the maximum likelihood algorithm. Chapter 16 introduces the ideas of random processes and their role in signal and image processing. Again the focus is on those processes needed to analyze noise in x-ray imaging. A student with a good grasp of Riemann integration should not have difficulty with the material in these chapters.